THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 52, No. 10, December, 1973
Printed in U.S.A.

Error Rates of Digital Signals in Charge Transfer Devices

By K. K. THORNBER

(Manuscript received June 21, 1973)

We calculate the probability of error in detecting digital signals transferred through a charge transfer device in the presence of incomplete charge transfer, random noise in the device, and detection uncertainty in the detector. The coefficient of incomplete charge transfer is assumed to be independent of charge-packet size, and both the device noise and detector noise are assumed to be Gaussian. Error probabilities for two-level and four-level codes are computed for the cases of both simple static and optimum dynamic detection. For rms detection voltage level fluctuations V_d of the order of tenths of volts (much larger than the random noise fluctuations in the device), a very rapid increase in error probability (from $\approx 10^{-20}$ to $\approx 10^{-5}$) is found to occur for a very small (20 percent) change in V_d. This indicates that detection level fluctuations will have to be held down to a few hundred millivolts at most. To achieve equal error rates with an error probability of about 10^{-14}, V_d for the detection of four-level codes will have to be about 3.5 times smaller *than for two-level codes. Comparison of error probabilities under static and dynamic detection shows that in CTD's improved detection has a greater potential for reducing error rates than improved coding.*

I. INTRODUCTION

As a packet of charge is transferred through a charge transfer device (CTD), the size of the packet is altered owing to effects of incomplete transfer[1,2] and noise.[3–9] At the output the size of each packet is measured and, depending on its size, a decision is made as to the initial size of the packet. Usually the decision will be correct. However, occasionally the cumulative effects of incomplete transfer and noise will result in a sufficiently distorted charge packet that an error will be made. It is the purpose of this paper to calculate the probability of making such a detection error. When this probability is multiplied by the rate of detection (the clock frequency), we obtain

the error rate for a single device. Multiplying by the number of devices of interest in the storage unit or in the processing unit, we obtain the total error rate, a very useful quantity for evaluating digital systems. (By "detection," we include regeneration; by a "single device," we mean a single unregenerated line of transfer elements.)

To calculate the probability of detection error we assume that the effect of incomplete transfer on the signal can be treated in terms of the usual small signal analysis.[10,11] (The coefficient of incomplete charge transfer, α, is assumed to be constant, independent of the size of the signal.) Charge gain or loss because of leakage current is assumed to be sufficiently small that it can be ignored. The random noise which introduces fluctuations into the size of the charge packets is assumed to be Gaussian.[8] This is reasonable by the law of large numbers, since the size of a charge packet is typically 10^6 elementary charges. In the numerical calculations, only shot noise at the input and thermal noise induced during charge transfer are considered, as these are the most important sources of noise in good devices.[8] In addition, the detection levels are assumed to fluctuate with Gaussian statistics. This simulates (*i*) the fluctuation in detection levels from device to device, (*ii*) the uncertainty in the location of the boundary between two decision regions, (*iii*) the uncertainty introduced from nonideal regeneration, and (*iv*) the fluctuations induced by the coupling of the clock lines to the output. In a future paper, we plan to treat several of these effects more carefully.

For our numerical work we take the position that probabilities of error of about 10^{-14} are of greatest interest. Values much higher would necessitate more-often-than-daily correction of a multimegabit store. Attention is focused on how large a fluctuation can be tolerated in the detection levels, so that the probability of error is in this region for the cases of two-level and four-level digital codes. In addition, the error probability is also examined as a function of the number of charge transfers. Similar calculations are made for the theoretically minimum possible error rate, which can be obtained using a dynamic detection scheme.[12] Comparison of this absolutely minimum error probability with the error probability obtained using conventional (static) detection suggests that a substantial improvement in error rate is possible using dynamic rather than static detection levels.[12]

II. PROBABILITY OF ERROR

In previous work,[10,12] it has been shown that in the absence of noise, after $(n + 1)$ transfers, each characterized by a coefficient of in-

complete transfer α, a charge packet of some initial size Q_i has at the output a size $Q(i)$ given by

$$Q(i) = (1 - \alpha)^{n+1} Q_i + Q_B, \tag{1}$$

where

$$Q_B = (1 - \alpha)^{n+1} \sum_{N=1}^{\infty} \binom{N+n}{N} \alpha^N Q_N, \tag{2}$$

and where Q_N is the initial size of the Nth packet preceding Q_i. In the presence of noise, the probability $P(Q - Q(i))dQ$ that the observed size Q of the packet is $Q(i)$ to within dQ is given by

$$P[Q - Q(i)]dQ = \exp\{-[Q - Q(i)]^2/(2\Delta Q^2)\}/(2\pi\Delta Q^2)^{\frac{1}{2}} dQ, \tag{3}$$

where ΔQ^2 is the mean-square fluctuation in the size of the charge packet at the output resulting from noise (see Appendix A). If the range of Q over which the packet will be detected as Q_i is given by $Q_i^- < Q < Q_i^+$, then P_i, the probability of error in detecting a specific $Q(i)$ packet, is

$$P_i = \int_{-\infty}^{Q_i^-} P[Q - Q(i)]dQ + \int_{Q_i^+}^{\infty} P[Q - Q(i)]dQ. \tag{4}$$

To determine error probability, P_i must be averaged over all possible $Q(i)$ for each i.

The quantities Q_i^- and Q_i^+ can be readily determined by rewriting (1) and (2) in the form

$$Q(i) = \bar{Q} + (Q_i - \bar{Q})(1 - \alpha)^{n+1} + Q_B', \tag{5}$$

where

$$Q_B' = (1 - \alpha)^{n+1} \sum_{N=1}^{\infty} \binom{N+n}{N} \alpha^N (Q_N - \bar{Q}). \tag{6}$$

In (5) and (6), $\bar{Q}$ is the (time) average size of a charge packet. (For example, if two packet sizes, Q_1 and Q_0, are used equally frequently in a two-level digital code, then $\bar{Q} = (Q_1 + Q_0)/2$.) If now we average eq. (5) over all possible preceding sequences of packets, then we obtain for $\langle Q(i) \rangle$, the average size of $Q(i)$,

$$\langle Q(i) \rangle = \bar{Q} + (Q_i - \bar{Q})(1 - \alpha)^{n+1}, \tag{7}$$

since the average $\bar{Q}_B'$ of Q_B' is zero. (Note that $\bar{Q}_N = \bar{Q}$ by definition.) The deviation of $Q(i)$ from $\langle Q(i) \rangle$ is simply Q_B', independent of i. $[Q(i) - \langle Q(i) \rangle = Q_B'.]$ By extending the results of a previous treatment[12] of two-level coding to the multilevel coding considered here, it follows at once (see Appendix B) that the theoretically minimum

possible error rates will be achieved for Q_i^- and Q_i^+ given by

$$Q_i^- = \frac{\langle Q(i)\rangle + \langle Q(i-1)\rangle}{2} + Q_B' \tag{8}$$

and

$$Q_i^+ = \frac{\langle Q(i)\rangle + \langle Q(i+1)\rangle}{2} + Q_B'. \tag{9}$$

(Note: $Q_{i+1}^- = Q_i^+$. For an M level system, $i = 0, 1, \cdots, M-1$. For completeness we define $Q(-1) \equiv -\infty$ and $Q(M) \equiv +\infty$.) These results apply even if the coding levels are not equally spaced. However, it should be obvious that if each size packet is used equally frequently, then equally spaced levels will result in the least probability of error.

In previous work[12] we have referred to the detection scheme which utilizes detection levels determined by Q_B' (that is, by the preceding signal) as a dynamic detection scheme. In other words, by subtracting out the incompletely transferred portion from the preceding signal prior to *each* detection (achievable under noiseless conditions), we can select detection regions which null out the scatter in the signal-charge size induced by incomplete transfer. Since random noise cannot be nulled out, a lower limit is placed on the error probability.

Using (8) and (9), we now compute the minimum error probability $P_{\min}$ of a single detection and average this over all possible preceding signals to obtain the minimum error probability $\langle P_{\min}\rangle$. Let p_i be the relative average frequency with which charge packets of initial size Q_i are used in the code. Then using (4) we may write

$$\begin{aligned} P_{\min} &= \sum_{i=0}^{M-1} p_i P_i \\ &= \sum_{i=0}^{M-1} p_i \left(\int_{-\infty}^{Q_i^- - Q(i)} P(Q)dQ + \int_{Q_i^+ - Q(i)}^{\infty} P(Q)dQ \right). \end{aligned} \tag{10}$$

If we note that $[Q_i^- - Q(i)] = -[\langle Q(i)\rangle - \langle Q(i-1)\rangle]/2$ and that $[Q_i^+ - Q(i)] = +[\langle Q(i+1)\rangle - \langle Q(i)\rangle]/2$, then $P_{\min}$ becomes

$$\begin{aligned} P_{\min} = \sum_{i=0}^{M-1} p_i \Bigg(&\int_{-\infty}^{-[\langle Q(i)\rangle - \langle Q(i-1)\rangle]/2} P(Q)dQ \\ &+ \int_{+[\langle Q(i+1)\rangle - \langle Q(i)\rangle]/2}^{+\infty} P(Q)dQ \Bigg). \end{aligned} \tag{11}$$

As mentioned in the preceding paragraph, $P_{\min}$ is independent of the foregoing charge packets. Thus $\langle P_{\min}\rangle = P_{\min}$. As $\langle P_{\min}\rangle$ is the minimal, or optimal, error probability, we will use it as a touchstone to compare other detection schemes.

Complete dynamic detection as discussed above is one extreme in detection. [Boonstra and Sangster[4] have operated a CTD utilizing a partial lowest order (in $n\alpha$) correction.] The other extreme in detection is to ignore completely the sequence of charge packets preceding the packet of interest and to attempt to detect without compensating for the accumulated hackground charge. Since the average of Q'_B is 0, one would then choose for Q_i^- and Q_i^+ the following

$$Q_i^- = [\langle Q(i)\rangle + \langle Q(i-1)\rangle]/2 \tag{12}$$

and

$$Q_i^+ = [\langle Q(i+1)\rangle + \langle Q(i)\rangle]/2. \tag{13}$$

In this case, the error probahility P associated with a specific detection event becomes

$$P = \sum_{i=0}^{M-1} p_i \left(\int_{-\infty}^{-[\langle Q(i)\rangle - \langle Q(i-1)\rangle]/2 - Q'_B} P(Q)dQ + \int_{+[\langle Q(i+1)\rangle - \langle Q(i)\rangle]/2 - Q'_B}^{\infty} P(Q)dQ \right). \tag{14}$$

To calculate $\langle P\rangle$, the average error probability, we must average (14) over all possible preceding signal sequences. Unlike $P_{\min}$, P is a function of the preceding sequence through Q'_B. In the remainder of this paper we shall focus attention on calculating $\langle P\rangle$.

III. NUMERICAL METHOD

Let us assume (i) that we are using a multilevel (M-level) code in which each size of charge packet is used equally frequently (so that $p_i = 1/M$), and (ii) that the levels of charge are equally spaced. Let $S^{\frac{1}{2}} \equiv [Q(i+1) - Q(i)]/2$. Then from (14), it follows that

$$\langle P\rangle = \frac{2M-2}{M} \int_{-\infty}^{-[S^{\frac{1}{2}}\cdot(1-\alpha)^{n+1} + Q'_B]} P(Q)dQ. \tag{15}$$

[Note: If n and α are such that $|Q'_B| > S^{\frac{1}{2}}$ for some sequences of charge packets, then errors are made with this detection scheme even in the ahsence of noise. Thus using the detection scheme characterized by the Q_i^- and Q_i^+ given by eqs. (12) and (13), it is essential that n and α be such that $|Q'_B| < S^{\frac{1}{2}}$ for all possihle sequences of packets. Thus, $(S^{\frac{1}{2}} + Q'_B) > 0$, and $\langle P\rangle < 1$.]

For numerical calculations, it is expedient to use (3) to rewrite (15) as

$$\langle P\rangle = 2\left(1 - \frac{1}{M}\right) \int_{-\infty}^{-(S/N)^{\frac{1}{2}}(1+\alpha)^{n+1}(1+\Sigma)} e^{-x^2/2} \frac{dx}{\sqrt{2\pi}}, \tag{16}$$

where S/N, the signal-power-to-noise-power ratio, is given by

$$\mathrm{S/N} = \frac{\{[Q(i+1) - Q(i)]/2\}^2}{\Delta Q^2}, \tag{17}$$

and where Σ is given by

$$\Sigma = \sum_{N=1}^{\infty} \binom{N+n}{N} \alpha^N J_N. \tag{18}$$

In (18) each $J_N[= (Q_N - \bar{Q})/S^{\frac{1}{2}}]$ is a random variable, which for M even can take on the values $\pm 1, \pm 3, \pm 5, \cdots, \pm(M-1)$ with equal probability, and for M odd is $0, \pm 2, \cdots, +(M-1)$ again with equal probability. To evaluate (16) in this form it is now necessary to average the integral in (16) over all possible sequences $J_1, J_2, J_3, \cdots$.

In this paper, we focus attention on that range of n and α which will probably be of greatest device interest—$n\alpha \ll 1$. In this case, only the first few terms in Σ will contribute significantly to its total value. By "significantly," we mean, of course, that whether $J_N = +(M-1)$ or $J_N = -(M-1)$ for fixed $J_1, \cdots, J_{N-1}$ and $0 = J_{N+1} = J_{N+2} = \cdots$, makes an acceptably small (say, 0.1%) difference in the values of the integral in (16). Thus, we can proceed as follows. Evaluate the integral in (16) for J_1 equal to each of its possible values and $0 = J_2 = J_3 = \cdots$, sum, divide by M, and multiply by $2(1 - 1/M)$. This gives a first estimate of $\langle P \rangle$ which we call $\langle P \rangle_1$. Now again evaluate the integral in (16) for all possible pairs of J_1, J_2 with $0 = J_3 = J_4 = \cdots$, sum, divide by M^2, and multiply by $2(1 - 1/M)$. This gives a second estimate of $\langle P \rangle$ which we call $\langle P \rangle_2$. In general, $\langle P \rangle_2 > \langle P \rangle_1$. If $(\langle P \rangle_2 - \langle P \rangle_1)/\langle P_1 \rangle$ is within the desired accuracy, then we may stop here. If $\langle P \rangle_2$ differs significantly from $\langle P \rangle_1$, we calculate $\langle P \rangle_3$ in the obvious way and compare to $\langle P \rangle_2$, etc. For the numerical results presented in the next section, $\langle P \rangle_3$ is as far as it is necessary to calculate to obtain 0.1 percent accuracy. For $n\alpha \ll 1$, convergence is guaranteed.

Often knowledge of the error probability to within a factor of 2 is adequate for design purposes. Thus, computing can be greatly facilitated if use is made of the following result. If $A > 1$, then

$$D/2 < I(A) < D, \tag{19}$$

where

$$I(A) = \int_{-\infty}^{-A} e^{-x^2/2} dx \tag{20}$$

and

$$D = \exp(-A^2/2)/A. \tag{21}$$

In the following section, our results for error probability are somewhat high, as D has been used in place of $I(A)$ in evaluating (16).

IV. NUMERICAL RESULTS

We have calculated both the minimum error probability $\langle P_{\min}\rangle$ [eq. (11)] using a dynamic detection scheme [eqs. (8) and (9)] and the error probability $\langle P\rangle$ [eq. (15)] using a static detection scheme [eqs. (12) and (13)], both for two-level and four-level coding. In all cases, we have taken $\alpha = 10^{-3}$, storage capacitance $C = 0.1$ pF, detection capacitance $C_{DE} = 0.1$ pF (see Appendix A), $Q_0 = C\cdot(4$

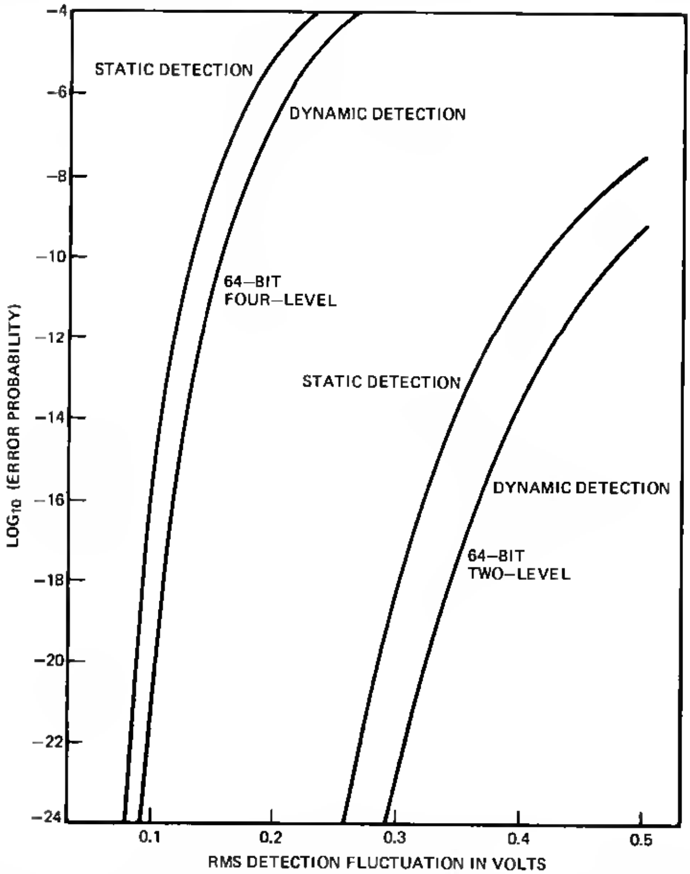

Fig. 1—Error probability as a function of the root-mean-square fluctuation in the detection level voltage for static and dynamic detection schemes of two-level and four-level coding in a 64-bit device.

volts), and $Q_{M-1} = C \cdot (10 \text{ volts})$. All calculations were carried to 0.1 percent.

In Fig. 1 we have plotted error probability $\langle P \rangle$ (static detection) and minimum error probability $\langle P_{\min} \rangle$ (dynamic detection) for two-level and four-level coding as a function of the root-mean-square detection-voltage fluctuation V_d. For the two-level results $n = 128$, and for the four-level results $n = 64$. Both these cases correspond to a 64-bit device. It is quite clear from Fig. 1 that to achieve an error probability of about 10^{-14}, for two-level coding $V_d < 0.345$ V, whereas for four-level coding $V_d < 0.105$ V. This means that, to be able to use four-level coding, we must have significantly better control of detection voltage fluctuation tban is necessary with two-level coding.

We might imagine that a trade-off could exist which would favor four-level coding. For example, only one-balf the number of transfer stages are needed with four levels as compared with two levels. Taking α inversely proportional[1,2] to C, for four levels we can double C and thereby cut α in balf relative to C and α for two levels. As α is reduced, the role of incomplete transfer is reduced as well. However, for $V_d = 0.35$ V, detection noise dominates the random noise. Thus S/N is practically unchanged as C is varied [see eq. (24) in Appendix A]. In addition, S/N for four levels is so small (≈ 8) that $\langle P \rangle$ goes only from $6.8 \cdot 10^{-3}$ for $\alpha = 10^{-3}$ to $4.3 \cdot 10^{-3}$ for $\alpha = 0.5 \cdot 10^{-3}$. Of course, for smaller V_d the change would be more drastic, as S/N would be larger. However, for smaller V_d, two-level operation is enhanced as well.

In Fig. 2 we have plotted the error probability of a two-level code as a function of tbe number of transfers for three different detection-level fluctuations for both static and dynamic detection schemes. In Fig. 3 we have plotted the same quantities for four-level coding and lower detection-level fluctuations. The striking superiority of dynamic detection over static detection is evident. (The dynamic curves are not actually flat; they increase somewhat in the region shown and much more rapidly for $n\alpha > 1$.)

V. CONCLUSIONS

In this paper we have derived expressions for the probability of error in detecting the size of charge packets carrying digital information in cbarge transfer devices. Effects of both random noise in the transfer device and detection noise at the detector were included. Error probabilities were computed and compared for common, static detection and optimum, dynamic detection of two types of coding

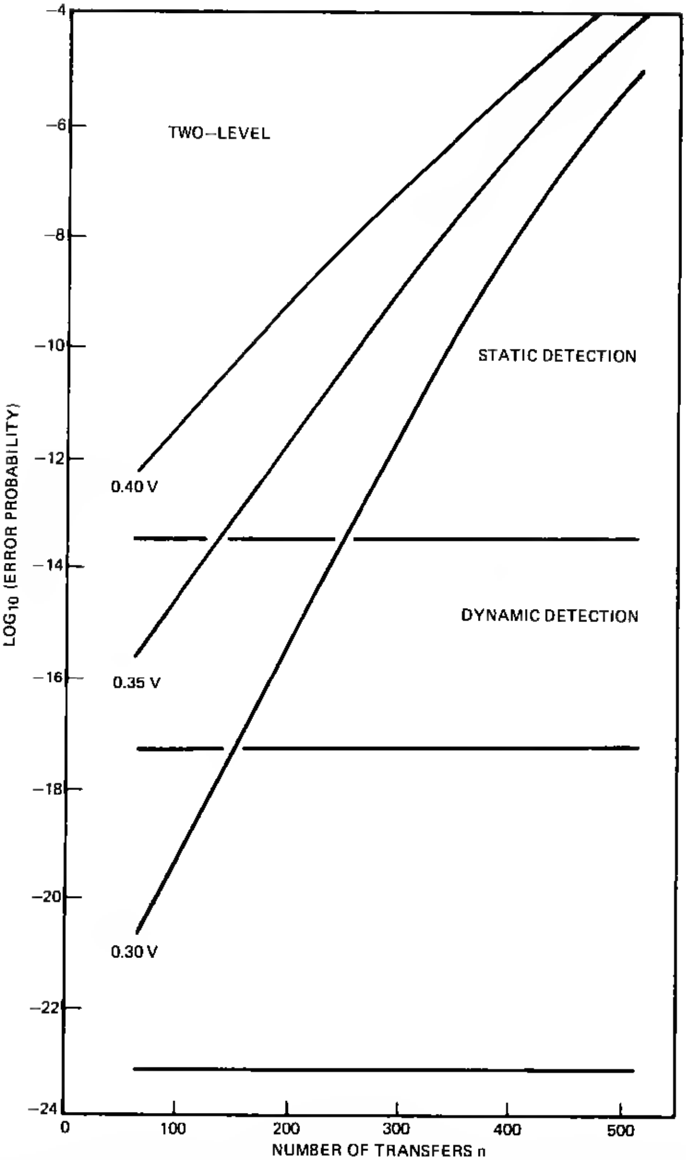

Fig. 2—Error probability as a function of the number of transfers n for static and dynamic detection of two-level coding for three values of root-mean-square detection voltage fluctuation (0.30, 0.35, and 0.40 V). For given n, the corresponding device is an $n/2$-bit device.

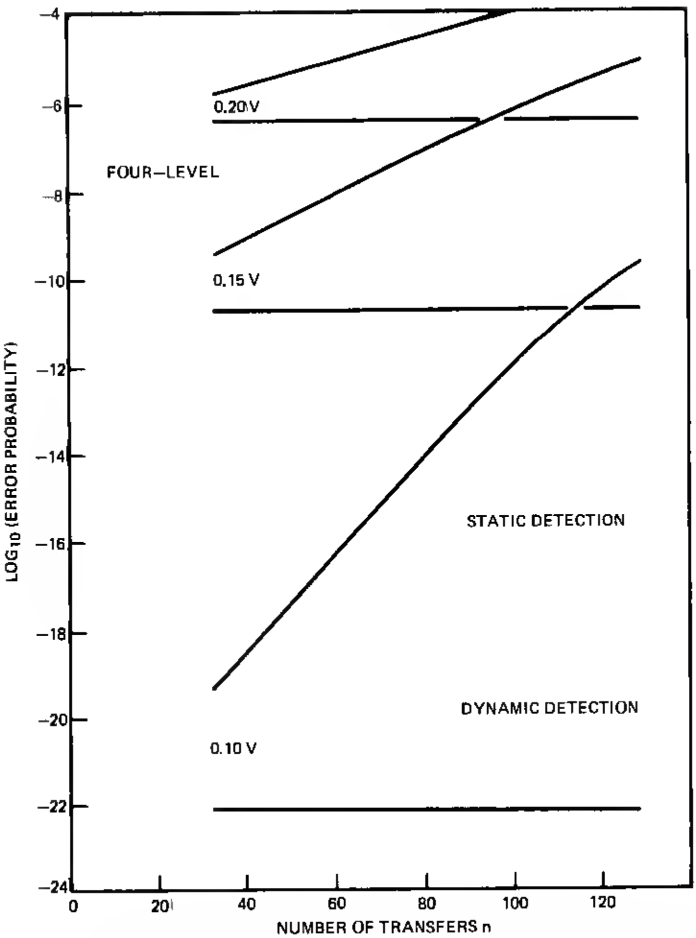

Fig. 3—Error probability as a function of the number of transfers n for static and dynamic detection of four-level coding for three values of root-mean-square detection voltage fluctuation (0.10, 0.15, and 0.20 V). For given n, the corresponding device is an n-bit device.

schemes. In the region of primary interest here (detection noise much larger than device noise), it was found that the error probability is a very sensitive function of detection noise, varying 20 orders of magnitude for a ± 20 percent change in the detection noise level. Also significant was the finding that, to achieve equivalent operational performance, the rms detection noise level in a device using a four-level code must be 3.5 times smaller than that in a device using a two-level

code. Thus, in designing circuits for digital signal detection, it will be necessary to focus primary attention on the detection level noise. This must be held to a few hundred millivolts at most. It was also shown how our dynamic detection scheme could maintain a very low error probability as the number of transfers, n, was greatly increased.

VI. ACKNOWLEDGMENTS

I wish to thank R. J. Strain and G. E. Smith for many helpful suggestions.

APPENDIX A

Noise

In general, we can write the mean-square noise charge, ΔQ_{rf}^2, resulting from random fluctuations in the form[8]

$$\Delta Q_{rf}^2 = \Delta Q_{\text{input}}^2 (1 - \alpha)^{2(n+1)} + \Delta Q_{SP}^2 H_{SP}(n+1) + 2\Delta Q_{TP}^2 H_{TP}(n+1), \quad (22)$$

where $\Delta Q_{\text{input}}^2$ is the input noise contribution, ΔQ_{SP}^2 is the storage process noise acquired by a single packet during a single clock period, ΔQ_{TP}^2 is the transfer process noise acquired by a single packet during a single charge transfer, $(1-\alpha)^{2(n+1)}$ is the (square of the) attenuation from input to output, $H_{SP}(n)$ is the compounding factor for storage process noise, and $H_{TP}(n)$ is the compounding factor for transfer process noise. A derivation of eq. (22) and a discussion of the various terms therein are treated elsewhere.[6-8] For our purposes ($n\alpha \ll 1$), it suffices to let $H_{SP}(n+1) = H_{TP}(n+1) = n+1$. (For $n\alpha \ll 1$, incomplete transfer of the noise can be ignored relative to the noise itself. Thus after $(n+1)$ transfers, the accumulated noise is just $(n+1)$ times the noise resulting from a single transfer.) We shall assume that $\Delta Q_{SP}^2 \ll \Delta Q_{TP}^2$ and set $\Delta Q_{SP}^2 = 0$. For shot noise at the input, $\Delta Q_{\text{input}}^2 = qQ$, where Q is the mean total signal charge $(Q_{M-1} - Q_0)$. For thermal noise, $\Delta Q_{TP}^2 = \frac{2}{3}kTC$. As it turns out, the exact details of ΔQ_{rf}^2 are not essential because these random effects turn out to be much smaller than the detection level fluctuations discussed below. However, if these detection level fluctuations can be reduced, then eq. (22) is quite important, especially in the region of $n\alpha \geqq 1$ where devices can operate using dynamic detection.

There are two equivalent ways in which detection level fluctuations can be included. The more systematic way is to use ΔQ_{rf}^2 in place of

ΔQ^2 in eqs. (3) and (4) and tben average over Q_i^- and Q_i^+ in (4) with the appropriate distribution for the detection level fluctuations. In this paper, we have restricted attention to a Gaussian distribution for these fluctuations. As the noise is also Gaussian, it follows from a straightforward integration that we can write eq. (4) in the form given in the text with

$$\Delta Q^2 = \Delta Q_{rf}^2 + \Delta Q_d^2, \tag{23}$$

where ΔQ_d^2 is the mean-square uncertainty of the detection level. [The second way is just to write (23) *a priori*.] Since some detection error will result from nonideal regeneration, once this can be more accurately simulated, a more careful analysis of detection uncertainty will be necessary.

The uncertainty ΔQ_d^2 is generated by an uncertainty V_d^2 in the detection voltage. Thus,

$$\Delta Q_d^2 = C_{DE}^2 V_d^2, \tag{24}$$

where C_{DE} is the capacitance of the detector. In our calculations, we have assumed that $C_{DE} = C$, wbere C is the elemental storage capacitance. If now $\Delta Q_d^2 \gg \Delta Q_{rf}^2$, then $S/N \approx V^2C^2/V_d^2C^2 = V^2/V_d^2$ independent of the capacitance. (Here V represents the signal voltage.) Thus, increasing C does *not* improve the signal-to-noise ratio (S/N) when detection noise exceeds random noise.

APPENDIX B

Minimum Error Probability

It is a very general result, rederived in a previous work,[12] that if $I(A)$ is defined by

$$I(A) = \int_{-\infty}^{-A} e^{-x^2/2} dx \tag{25}$$

and if tbe probability that $A < 0$ is 0, then

$$\langle I(A) \rangle \geqq I(\langle A \rangle). \tag{26}$$

Thus any detection scheme for which

$$\int_{-\infty}^{Q_i^-} P[Q - Q(i)] dQ = \int_{-\infty}^{Q_i^- - Q(i)} P(Q) dQ = \int_{-\infty}^{\langle Q_i^- - Q(i) \rangle} P(Q) dQ \tag{27}$$

for each i (and the corresponding equalities for $\int_{Q_i^+}^{\infty} \cdots dQ$) will result in the minima overall error probability. The choice given in eqs. (8) and (9) does this, as it makes $Q_i^- - Q(i)$ independent of the preceding

sequence of charge packets over which the average in (26) and (27) is taken.

APPENDIX C

Realizability of Dynamic Detection Scheme

Before considering the realizability of the scheme of dynamic detection discussed in the text, a point of clarification is necessary. One reason for developing dynamic detection here is to see just how low we can, in principle, make the error rate. This we have done assuming Gaussian noise, linear incomplete transfer, *and* complete knowledge of the preceding signal. If we relax the last assumption, we must take into account the fact that our detection of the preceding signal may not be perfect and, therefore, a higher error rate may in fact be the minimal rate possible physically. This problem is more difficult and will not be attempted here. What is important to distinguish, however, is the difference between "perfect" dynamic detection, which provides a minimum error rate below which one cannot hope to achieve, and the actual error rate when employing dynamic detection, which as we shall indicate below is not appreciably larger than the minimum rate under operating conditions of interest. With this in mind, let us proceed to a consideration of realizability.

In the absence of noise, the dynamic detection scheme is clearly realizable in principle. Knowledge of the preceding signal permits determining the background charge level Q_B (resulting from incomplete transfer) operationally using eq. (2). This permits placing the detection levels so that the size of the charge packet to be detected will lie midway between these detection levels. Under noiseless conditions, this permits error-free detection which, in turn, provides the signal history needed to determine Q_B for the next packet detection.

In the presence of noise, one may ask whether the dynamic detection scheme envisioned in Section II is truly realizable. If, for example, an error is made in detection, then the detection levels may be shifted far enough away from optimum so that for the next packet the error probability will be greatly increased. Fortunately, as the argument below suggests, if the probability of making a second error immediately following the first is small compared to unity, then the optimum (minimum) error probabilities presented in the text are only slightly increased (on the order of percents rather than order of magnitude).

Consider a two-level code and the dynamic detection scheme in which it is only necessary to adjust the detection levels for the first

preceding signal [$Q_B = \alpha(n+1)Q_1$, Q_1 = size of first preceding packet]. We desire the probability $P_c(i)$ that the ith packet is detected correctly. Clearly,

$$P_c(i+1) = P_{cc}(i+1|i)P_c(i) + P_{ce}(i+1|i)P_e(i), \tag{28}$$

where $P_e(i)$ [$= 1 - P_c(i)$] is the probability that the ith packet is detected incorrectly, $P_{cc}(i+1|i)$ is the probability that the $(i+1)$th packet is correctly detected given that the ith was also, and $P_{ce}(i+1|i)$ is the probability that the $(i+1)$th packet is correctly detected given that the ith was detected incorrectly. Noting that $P_c(i+1) = P_c(i) = P_c$, we can solve (28) for P_c, obtaining

$$P_c = [1 + P_{ec}(i+1|i)/P_{ce}(i+1|i)]^{-1}, \tag{29}$$

where $P_{ec} = 1 - P_{cc}$. The error probability $P_e(= 1 - P_c)$ which we seek is, therefore, given by

$$P_e = [1 + P_{ce}(i+1|i)/P_{ec}(i+1|i)]^{-1} \tag{30}$$

$$\approx \frac{P_{ec}}{P_{ce}} = \frac{P_{ec}}{1 - P_{ee}}. \tag{31}$$

Equation (31) follows if $P_{ec} \ll P_{ce}$, as will be the case for $P_e \ll 1$, which is the region of greatest interest. If now $P_{ee} < 0.1$, then P_e will differ from P_{ec} [calculated in the text, eq. (11), as $\langle P_{\min}\rangle$] by less than 10 percent, an insignificant change. Although we have not investigated P_{ee} in detail, it is clear that P_{ee} will be closer in size to $\langle P\rangle$ (eq. 15) corresponding to static detection rather than to $\langle P_{\min}\rangle$. However, what is important is that P_{ee} can be as large as one-half without increasing $\langle P_{\min}\rangle$ by more than a factor of 2. Thus, the $\langle P_{\min}\rangle$ calculated here are not expected to be overly optimistic so long as the detection level need only be corrected on the basis of just the first preceding signal. For the present, this is the situation of primary interest. It should be kept in mind, however, that it is the random noise and not the incomplete transfer which complicates dynamic detection. With sufficiently low noise, we can in principle greatly reduce incomplete-transfer distortion without appreciable propagation of detection errors even when the detection level must be corrected on the basis of many preceding signals.

REFERENCES

1. Berglund, C. N., Thornber, K. K., "Incomplete Transfer in Charge Transfer Devices," IEEE J. Solid-State Circuits, *SC-18* (1973), pp. 108–116. See Refs. 5 to 21 of this reference for a list of prior treatments of incomplete transfer.

2. Berglund, C. N., and Thornber, K. K. "Fundamental Comparison of Incomplete Charge Transfer in Charge Transfer Devices," B.S.T.J., *52*, No. 2 (February 1973), pp. 147–182.
3. van der Ziel, A., "Noise in Solid State Devices and Lasers," Proc. of the IEEE, *58* (1970), pp. 1178–1206.
4. Boonstra, L., and Sangster, F. L. J., "Progress on Bucket-Brigade Charge-Transfer Devices," 1972 IEEE Solid-State Conference, Digest of Technical Papers *15* (1972), pp. 140–1.
5. Tompsett, M. F., "Quantitative Effects of Interface States on the Performance of Charge-Coupled Devices," IEEE Transactions on Electron Devices, *ED-20* (1973), pp. 46–55.
6. Thornber, K. K., "Noise Suppression in Charge Transfer Devices," Proc. of the IEEE, *60*, pp. 1113–4.
7. Thornber, K. K., and Tompsett, M. F., "Spectral Density of Noise Generated in Charge Transfer Devices," IEEE Transactions on Electron Devices, *ED-20* (1973). p. 456.
8. Thornber, K. K. "Theory of Noise in Charge Transfer Devices," in preparation.
9. Carnes, J. E., and Kosonocky, W. F. "Noise Sources in Charge Coupled Devices," RCA Review, *33* (1972), pp. 327–343.
10. Joyce, W. B., and Bertram, W J., "Linearized Dispersion Relation and Green's Function for Discrete-Charge-Transfer Devices With Incomplete Transfer," B.S.T.J., *50*, No. 6 (July-August 1971), pp. 1741–1759.
11. Berglund, C. N., "Analog Performance Limitations of Charge Transfer Dynamic Shift Resisters," IEEE J. Solid-State Circuits, *SC-6* (December 1971), pp. 391–394.
12. Thornber, K. K., "Operational Limitations of Charge Transfer Devices," B.S.T.J., *52*, No. 9 (November 1973), pp. 1453–1482. The residual charge, Q_R, used in this reference is the same entity as the background charge, Q_B, used in the present paper.